EXERCISE – III
SUBJECTIVE QUESTIONS

1. Find the real values of x and y for which the following equation is satisfied $\frac{(1+i)x-2i}{3+i}+\frac{(2-3i)y+i}{3-i} = i$

2. Find the square root of
(i) 7 + 24 i

(ii) 4 + 3i

3. Find the modulus, argument and the principal argument of the complex numbers.

(a) $z = 1 + \cos\frac{18\pi}{25} + i \sin\frac{18\pi}{25}$

(b) $z = -2(\cos 30^\circ + i \sin 30^\circ)$

4. Interpret the following locii in $z \in C$.
(a) $1 < |z - 2i| < 3$

(b) $Im(z) \geq 1$

(c) $Arg(z - a) = \pi/3$ where $a = 3 + 4i$

5. If $|z - 2 + i| \leq 2$, then find the greatest and least value of $|z|$.

6. If $|z + 3| \leq 3$ then find minimum and maximum values of
(i) $|z|$ **(ii)** $|z - 1|$ **(iii)** $|z + 1|$

7. If O is origin and affixes of P, Q, R are respectively z, iz, z + iz. Locate the points on complex plane. If $\Delta PQR = 200$ then find
(i) $|z|$ **(ii)** sides of quadrilateral OPRQ

8. If $|z_1| = |z_2| = ... = |z_n| = 1$ then show that
(i) $\bar{z}_1 = \frac{1}{z_1}$

(ii) $|z_1 + z_2 + ... + z_n| = \left|\frac{1}{z_1}+\frac{1}{z_2}+......+\frac{1}{z_n}\right|$.
And hence interpret that the centroid of polygon with 2n vertices $z_1, z_2 .... z_n, \frac{1}{z_1}, \frac{1}{z_2}, ...\frac{1}{z_n}$ (need not be in order) lies on real axis.

9. Plot the region represented by $Re(z) \leq 2$, $Im(z) \leq 2$ and $\frac{\pi}{8} \leq arg(z) \leq \frac{3\pi}{8}$.

10. If n is a positive integer, prove the following
(i) $(1 + \cos\theta + i \sin\theta)^n + (1 + \cos\theta - i\sin\theta)^n$
$= 2^{n+1}\cos^n\frac{\theta}{2}\cos\frac{n\theta}{2}$.

(ii) $(1 + i)^n + (1 - i)^n = 2^{\frac{n}{2}+1}.\cos\frac{n\pi}{4}$

11. Solve $(z - 1)^4 - 16 = 0$. Find sum of roots. Locate roots, sum of roots and centroid of polygon formed by roots in complex plane.

12. Find the values(s) of the following
(i) $\left(\frac{1}{2}+\frac{\sqrt{-3}}{2}\right)^3$

(ii) $\left(\frac{1}{2}+\frac{\sqrt{-3}}{2}\right)^{3/4}$
Hence find continued product if two or more distinct values exists.

13. Let I : $Arg\left(\frac{z-8i}{z+6}\right) = \pm\frac{\pi}{2}$ II : $Re\left(\frac{z-8i}{z+6}\right) = 0$
Show that locus of z in I or II lies on $x^2+y^2+6x - 8y=0$
Hence show that locus of z can also be represented by $\frac{z-8i}{z+6}+\frac{\bar{z}-8i}{\bar{z}+6} = 0$. Further if locus of z is expressed as $|z + 3 - 4i| = R$, then find R.

14. If α is imaginary n^{th} $(n \geq 3)$ root of unity then show that $\sum_{r=1}^{n-1}(n-r)\alpha^r = \frac{n\alpha}{1-\alpha}$. Hence deduce that $\sum_{r=1}^{n-1}(n-r)\sin\frac{2\pi r}{n} = \frac{n}{2}\cot\frac{\pi}{n}$

15. Find the real values of the parameter 'a' for which at least one complex number $z = x + iy$ satisfies both the equality $|z - ai| = a + 4$ and the inequality $|z - 2| < 1$.

16. If α, β, γ are roots of $x^3 - 3x^2 + 3x + 7 = 0$ and ω is imaginary cube root of unity, then find the value of $\frac{\alpha-1}{\beta-1}+\frac{\beta-1}{\gamma-1}+\frac{\gamma-1}{\alpha-1}$.

17. Among the complex numbers z satisfying the condition $|z + 3 - \sqrt{3}\,i| = \sqrt{3}$, find the number having the least positive argument.

18. Given $z_1 + z_2 + z_3 = A$, $z_1 + z_2\,\omega + z_3\,\omega^2 = B$, $z_1 + z_2\,\omega^2 + z_3\,\omega = C$, where ω is cube root of unity,

(a) express z_1, z_2, z_3 in terms of A, B, C.

(b) prove that, $|A|^2+|B|^2+|C|^2=(|z_1|^2+|z_2|^2+|z_3|^2)$.

(c) prove that $A^3 + B^3 + C^3 - 3ABC = 27z_1z_2z_3$

19. Prove that, with regard to the quadratic equation $z^2 + (p + ip')\,z + q + iq' = 0$; where p, p′, q, q′ are all real.

(a) If the equation has one real root then $q'^2 - pp'\,q' + pq'^2 = 0$.

(b) If the equation has two equal roots then $p^2 - p'^2 = 4q$ & $pp' = 2q'$.
State whether these equal roots are real or complex.

20. Simplify and express the result in the form of a+bi

(a) $\left(\frac{1+2i}{2+i}\right)^2$

(b) $-(i\,(9 + 6i)\,(2 - i)^{-1})$

(c) $\left(\frac{4i^3-i}{2i+1}\right)^2$

(d) $\frac{3+2i}{2-5i}+\frac{3-2i}{2+5i}$

(e) $\frac{(2+i)^2}{2-i}-\frac{(2-i)^2}{2+i}$

21. Given that $x, y \in R$ solve

(a) $(x + 2y) + i(2x - 3y) = 5 - 4i$

(b) $(x + iy) + (7 - 5i) = 9 + 4i$

(c) $x^2 - y^2 - i(2x + y) = 2i$

(d) $(2 + 3i)x^2 - (3 - 2i)\,y = 2x - 3y + 5i$

(e) $4x^2+3xy+(2xy-3x^2)i = 4y^2-(x^2/2) + (3xy-2y^2)\,i$

22. Show that all the roots of the equation $a_1z^3 + a_2z^2 + a_3z + a_4 = 3$, where $|a_i| \le 1$, $i = 1, 2, 3, 4$ lie outside the circle with centre origin and radius 2/3.

23. If a & b are real numbers between 0 & 1 such that the points $z_1 = a + i$, $z_2 = 1 + bi$ & $z_3 = 0$ from an equilateral triangle, then find the values of 'a' and 'b'.

24. (a) Find all non-zero complex numbers Z satisfying $\overline{Z} = iZ^2$.

(b) If the complex numbers $z_1, z_2, ...z_n$ lie on te unit circle $|z| = 1$ then show that $|z_1 + z_2 + ... + z_n| = |z_1^{-1} + z_2^{-1} + ... + z_n^{-1}|$.

25. Find the Cartesian equations of the locus of 'z' in the complex plane satisfying $|z - 4| + |z + 4| = 16$.

26. If ω is an imaginary cube root of unity then prove that

(a) $(1 + \omega - \omega^2)^3 - (1 - \omega + \omega^2)^3 = 0$

(b) $(1 - \omega + \omega^2)^5 + (1 + \omega - \omega^2)^5 = 32$

(c) If ω is the cube root of unity, Find the value of $(1 + 5\omega^2 + \omega^4)\,(1 + 5\omega^4 + \omega^2)\,(5\omega^3 + \omega + \omega^2)$.

27. Locate the points representing the complex number z on the Argand plane

(a) $|z + 1 - 2i| = \sqrt{7}$

(b) $|z - 1|^2 + |z + 1|^2 = 4$

(c) $\left|\frac{z-3}{z+3}\right| = 3$ **(d)** $|z - 3| = |z - 6|$

28. Find the modulus, argument and the principal argument of the complex numbers.

(i) $6(\cos 310^\circ - i \sin 310^\circ)$

(ii) $-2(\cos 30^\circ + i \sin 30^\circ)$

(iii) $\frac{2+i}{4i+(1+i)^2}$

29. Prove that identity,

$|1 - z_1 \bar{z}_2|^2 - |z_1 - z_2|^2 = (1 - |z_1|^2)(1 - |z_2|^2)$

30. If ω is a cube root of unity, prove that

(i) $(1 + \omega - \omega^2)^3 - (1 - \omega + \omega^2)^3 = 0$

(ii) $\frac{a + b\omega + c\omega^2}{c + a\omega + b\omega^2} = \omega^2$

(iii) $(1 - \omega)(1 - \omega^2)(1 - \omega^4)(1 - \omega^8) = 9$

31. If $x = a + b$; $y = a\omega + b\omega^2$; $z = a\omega^2 + b\omega$, show that

(i) $xyz = a^3 + b^3$

(ii) $x^2 + y^2 + z^2 = 6ab$

(iii) $x^3 + y^3 + z^3 = 3(a^3 + b^3)$